Comparative Spatial Segregation Analytics

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"All of the segregation indexes have in common the assumption that segregation can be measured without regard to the spatial patterns of white and nonwhite residence in a city" (Duncan and Duncan, 1955).

Abstract

Comparative segregation analysis holds the potential to provide rich insights into urban socio-spatial dynamics. However, comparisons of the levels of segregation between two, or more, cities at the same point in time complicated by different spatial contexts. The extent to which differences in segregation between two cities is due to differences in spatial structure or to differences in composition remains an open question. This paper develops a framework to disentangle the contributions of spatial structure and ethnic composition in carrying out comparative segregation analysis. The approach uses spatially explicit counterfactuals embedded in a Shapley decomposition. We illustrate this approach in a case study of the 50 largest metropolitan statistical areas in the U.S.

1 Introduction

Comparison of the levels of segregation across US cities is a popular pursuit in both academia[1] and in the popular press.[2] Most often, these comparisons follow a similar strategy involving the calculation of an index of segregation for a collection of cities at one point in time, followed by a ranking of the values for the index.

The resulting rankings invariably garner widespread attention. Yet, from a methodological point of view, they also raise a number of questions. The ordinal

[1] http://www.censusscope.org/us/print_rank_dissimilarity_white_black.html

[2] https://www.washingtonpost.com/graphics/2018/national/segregation-us-cities/?noredirect=on&utm_term=.b8e434f29177

nature of these summaries is often emphasized. Finding that, in 2000, Chicago ranked 6th[1] while Newark ranked 7th conveys a different impression than knowing the former had a dissimilarity index of 83.6 relative to the latter's index of 83.4. The question of whether these differences are significant often goes unasked, and therefore, unanswered. This is curiously distinct from much quantitative social science research where questions of inference are central to the investigation. One of the main reasons for the descriptive orientation of much of the segregation literature is the limited amount of work developing inferential approaches.

Existing work on inference has adopted an analytical approach and, based on assumptions about the data generating process, the distribution for a particular index is analyzed and its sampling distribution derived (Allen et al., 2015). A key issue in the literature is the so called small unit problem which concerns an upward bias in segregation indices when the sampling unit is small and thus contains a small number of people. Because most indices are functions of proportions, they can suffer from a small sample problem due to large sampling variance because the denominators are small. Adjustments to the upward bias have been identified in the literature but have limitations due to their reliance on asymptotic reasoning.

Alongside the general neglect of questions of inference, and the limited amount of work on inferential frameworks for segregation, the quote by Duncan suggests these indices may only be capturing part of the complex nature of urban segregation. In modern parlance, most segregation indices are "locationally invariant". That is, they are insensitive to the spatial distribution of the group shares across the enumeration units in a city.

While this has been recognized for over half a century, we argue spatial questions take on increasing importance when the focus is on comparative segregation analysis. The complexities, and differences, in the spatial structures of Chicago and Newark pose challenges to comparative investigations, yet these remain ignored. The same challenges are likely to hold in any comparisons of different cities at the same point in time, the same city at two points in time, or the comparative dynamics of segregation between two different cities and time periods.

In this paper we consider comparative segregation analysis from a spatially explicit perspective to make several contributions. We propose a framework for disentangling the roles of varying spatial structure and composition when carrying out comparative segregation analysis. Our framework is based on a decomposition of differences in segregation between two cities, or the same city at two points over time into spatial and attribute contributions. The approach relies on novel counterfactual distributions for the comparisons cities together with a Shapley value decomposition defined on these counterfactuals. We also provide empirical insights on the magnitude of these components across 50 metropolitan areas in the US over the period 2000-2010.

The remainder of the paper is organized as follows. We first revisit the literature on comparative segregation and examine the complications that spatial structure and effects pose for such analyses. In section 3, we present our framework for comparative segregation analysis that is designed to address some of these issues. We then provide an empirical illustration of our framework in section 4. The paper concludes with a summary of key points and suggestions for future areas of research.

2 Comparing Measures of Segregation

Calculating segregation measures and drawing comparisons between cities is among the oldest traditions in urban social science. Indeed, the "concentric zones model" of the fabled Chicago school, to which many trace the genesis of contemporary urban studies, was constructed and defended through the comparative analysis of segregation in American cities. Focusing on Chicago, Detroit, Manhattan, Cleveland, Philadelphia, and Pittsburgh, Burgess (1928) famously counted the number of neighborhoods in which different minority groups comprised more than 10% of the population. Using his admittedly coarse measure, he then compared the cities with one another, leveraging the results to argue that similar segregation structures appeared in each of them, leading to validation for the concentric zones model. In the century that followed, a massive literature appeared that focused on the measurement of segregation, much of which followed Burgess's original recipe: select a particular segregation measure, calculate the measure for a variety of cities, and use the resulting statistics to compare and contrast the patterns found in each city. Despite the simplicity of this recipe, there is considerable breadth and depth to this literature.

Much of the segregation literature focuses on developing improvements to statistical measurement techniques (Massey, 1978b; Wong, 1993; Massey et al., 1996; Wong, 1999; Reardon and Firebaugh, 2002; Wong, 2003, 2004; Dawkins, 2004; Reardon and O'Sullivan, 2004; Wong, 2005; Sean F. Reardon et al., 2008; Chodrow, 2017), while a parallel body applies these metrics to the study of gender, ethnic, racial, occupational, educational, income, and other forms of segregation (Mare and Bruch, 2011). For decades, these two strains have fed off one another, with empirical studies revealing undesirable properties of common segregation indices, and statistical work proposing alternative techniques or corrections to account for the identified shortcomings. Despite a constant stream of incremental improvements, and dozens of segregation indices proposed and applied throughout the literature, there remains considerable room for improvement in methods designed for segregation analysis, particularly from the perspective of comparative frameworks. Put differently, while there has been vast improvement in the theoretical and computational measurement of segregation, the past century has seen almost no innovation that is able to overcome the "problems of inter-urban comparative work that arise because of the nature of available census data sets" (Johnston, 1981, p.246). Herein we review these problems and the ways in which various scholarship has tried to address them.

Existing Examples

Comparisons Over Space

In the canon of comparative segregation studies, the most common methodological technique is for researchers to choose and defend the use of a particular segregation index, calculate index values for a set of cities or regions, and rank and compare the resulting values describing the ordinal structure across cities. Thereafter, researchers sometimes examine how these ordinal rankings differ for alternative indices. For decades, scholars have deployed these descriptive methods successfully to compare residential segregation in a wide range of contexts, cultures, and time periods. Both canonical and recent work has examined segregation by race and class in American cities thoroughly (Clark, 1986a; Ihlanfeldt and Scafidi, 2002, 2004; Brinegar and Leonard, 2008; Hwang, 2015; Wang et al., 2018). But scholarship is by no means limited to American cities or social constructs. Elsewhere, researchers have compared segregation measurements between global cities (Harsman and Quigley, 1992; Marcińczak et al., 2015; Musterd et al., 2017), between countries (Goering, 1993; Johnston et al., 2007), and within cities in countries across the globe (Morgan, 1975; Owusu and Agyei-Mensah, 2011; Wang and Li, 2016). Neither are place-based comparisons limited necessarily to analysis of residential segregation. A large body of work in sociology and labor studies examines occupational segregation by race and gender, and how those patterns compare across countries and/or labor markets (Blackburn et al., 1993), and while these works are conceptually distinct, their formula for comparative analysis is identical.

Comparative studies of this variety are useful because they permit observations such as "in general there is less segregation in Australia and New Zealand" than in Canada, the United Kingdom and the United States (Johnston et al., 2007, p.713), and these patterns can be further analyzed by the differing social contexts of each country, or used to develop a policy agenda, etc. But the simplicity of this analytical technique and the resulting rankings masks a crucial uncertainty because inter-urban comparisons are never truly "apples-to-apples". It is impossible to compare segregation in City A versus City B while accounting appropriately for the idiosyncratic differences between them in size, scale, and configuration.

Critiques in comparative segregation research often arise over concerns about data quality and measurement approaches. Chief among the criticisms is that segregation measurements are sensitive to (at least) two critical features of urban areas beyond control of the analyst. First, since indices operate on population ratios, they are notably sensitive to the relative size of different population groups in each city. Small shares of minority populations can inflate widely used measures like the index of dissimilarity (Cortese et al., 1976; Clark, 1986b; Massey, 1978a; Reardon and O'Sullivan, 2004). Second, while residential segregation is a multiscalar phenomenon whose smallest scale manifests at the housing-unit level, the census data used to calculate segregation measurements necessarily relies on aggregations to larger polygons to protect confidentiality. As a result, "all measures of spatial and aspatial segregation that rely on population counts aggregated within subareas are sensitive to the definitions of the boundaries of these spatial subareas" (Reardon and O'Sullivan, 2004)[p.124]. That is, segregation indices are significantly affected by the size and shape of the census tracts (or other spatial units) that serve as the basis of such measures (Jakubs and Jakubus, 1981; Massey, 1978b). To overcome issues related to census enumeration units, Reardon and O'Sullivan (2004) interpolate census blocks to a regular grid so that spatial units approximate a continuous surface, and several others have adopted this technique (Lee et al., 2008; Reardon et al., 2006) in the literature. While this method skirts issues of census boundary configuration, kernel-based interpolation of this variety relies on what may be highly questionable assumptions about population density, and explicitly ignores important physical features like impassable terrain or uneven development. As a result, the population surfaces are often inaccurate, raising questions about the validity of segregation measures generated by these techniques.

Apart from technical issues inherent in the properties of particular indices and the applicability of available data, methods for comparative analysis still leave much to be desired since, as Clark (1986b) points out, the conceptual distinction between indices can lead to significantly different interpretations in applied settings. The difference in segregation between City A and City B may look trivial when measured with the index of isolation, but appear significantly larger when measured with the Gini index. Current techniques leave no recourse for this problem other than argumentation regarding which index is the superior, trustworthy measure– but even in the case of perfectly unbiased indices, methodological impediments remain.

In their classic study, Massey et al. (1996) describe five conceptual dimensions of segregation they term evenness, exposure, concentration, centralization, and clustering, and while there is debate over whether these represent the "true" dimensions of residential segregation, there is nonetheless agreement that multiple dimensions are worthy of consideration. Thus, dozens of segregation indices persist in the literature, thanks in part to their desirable sensitivity to various different dimensions. In applied comparative research, however, differential sensitivity can by definition lead to ambiguous results. In problematic cases, segregation indices disagree by wide margins, as discussed by Clark (1986a)[p.97] who shows that "Baltimore (Table 1) was almost twice as segregated as San Jose on the dissimilarity index in 1970, but the exposure index suggested that while Baltimore was substantially segregated, San Jose was not". Explaining the gap between these measures for the two cities presents an interesting avenue for further study, but also an impasse for statistical comparative work. To our knowledge, no existing quantitative techniques are capable of analyzing whether the segregation measures for each city are significantly different in either semantic or statistical terms.

Comparisons Over Time

A natural extension of comparative segregation analysis is the inclusion of time as an important dimension. Incorporating temporality into the study of urban segregation typically assumes one of three flavors; A large body of work examines the experience of individual households, and whether minority members are able to escape segregated neighborhoods in successive generations (Bischoff and Reardon, 2013). This work grows from the life course tradition in sociology to address questions pertaining to the long-term experience of neighborhood and community realized by members of minority groups (McAvay, 2018). While this literature sheds considerable light onto the prevalence and persistence of intergenerational segregation, it typically takes segregation measures as given, and focuses the analysis on migration patterns that expose families higher or lower levels of urban segregation. The emphasis here is less on the measurement of segregation and more on the residential mobility patterns that bring individuals into contact with segregation, and thus is less useful for comparative work.

Another currently very active area of research in the urban studies focuses on measuring segregation as a function of daily time. This is an extension of research on time geography and seeks to incorporate temporal variation in the experiences of individuals as they move throughout an urban activity space, recognizing that the *experience* of segregation expands well beyond the residential neighborhood and into the employment, leisure, and entertainment spaces that people move through and inhabit throughout the day (Hägerstraand, 2005; van Ham and Tammaru, 2016; Kwan, 2015; Park and Kwan, 2018). As such, the activity space literature attempts to capture the dynamic and composite experience of segregation resulting from the social interactions that occur in these spaces as population homogeneity fluctuates naturally throughout the course of a typical day (Wong and Shaw, 2011; Kwan, 2015; Wang and Li, 2016; Wang et al., 2018; Zhang et al., 2018). Work on activity-space segregation is appearing more frequently in recent years, and nascent but growing literature has revealed an important dimension to the ways in which segregation is *realized* through daily routine.

Despite its utility for showing how measures fluctuate in space over time, this literature engages less with temporal *comparisons*, and more with the ways in which segregation levels can be averaged more reliably over the course of a day. For this reason, some scholars have argued that it is preferable to treat activity space as a measure of spatial context as opposed to a proper temporal consideration of segregation (Fowler et al., 2016). We agree with this logic and find the activity space literature less informative for comparative segregation analytics.

For comparative analysis, the most relevant area of research seeks to examine how a given segregation index evolves in a single place over time. This includes the calculation of a particular segregation index at several cross-sectional periods, then examining overall trends in index values over time and/or between discrete time-steps (usually decennial). Here, scholars seek to address questions of how the racial and ethnic mix within a system of neighborhoods is evolving over time, and whether shifting demographic migration patterns lead to more or less integration. Unlike the temporal methods discussed above, which take the perspective of individuals or groups and examine how mobility exposes them to different spatial contexts, this work takes the perspective of places and examines how migration and residential selection affect segregation measures at the city level. For decades, this has been a large and active area of urban research, but has become increasingly so in recent years as more data become available. Again, researchers follow the pattern of choosing a study area, computing segregation indices at several points in time, and examining the linear trend (Lieberson and Carter, 1982; Charles, 2003; Bailey, 2012; De la Roca et al., 2014; Intrator et al., 2016).

Temporal comparative analysis provides a unique window into the dynamic structure of segregation and the ways in which urban areas are evolving. Comparisons over time help us understand the paths that cities follow, and whether they trend toward integration or separation, though these analyses too suffer a variety of drawbacks. Among the chief criticisms of temporal comparisons is that they necessarily rely on decennial census data, which captures "just one part of the picture, applying only to the population present and captured at both time points" (Bailey, 2012, p.709). Decennial data are severely limited for segregation analysis because they fail to capture the volatility in metropolitan housing markets that occurs over a 10 year period. Apart from issues of data concurrency, temporal comparisons suffer other shortcomings as well. As with place-based comparisons discussed above, temporal comparisons rely on census data, which are retabulated each year according to changes in population density. This means that in theory, the segregation levels measured in a particular city could change over time even if population ratios and spatial allocation remained constant, but tabulation blocks were redrawn between the two decades (Jakubs and Jakubus, 1981; Massey, 1978b; Reardon and O'Sullivan, 2004; Logan et al., 2014). Finally, as discussed earlier even in the case of perfect data and stable tabulation units, there is no statistical framework for assessing whether the difference between two temporal comparisons is meaningful.

Comparisons Over Space-Time

A final mode of comparison in the field of segregation analytics is that between places over time. As the most data intensive, this is naturally the smallest of the three reviewed fields, with a far more modest collection of work taking up the challenge. Nevertheless, there are a number of papers that analyze segregation levels over time in several places, most of which follow a combination of the portions reviewed prior; Purely place-based comparisons are, necessarily, between point estimates of particular segregation indices. Time-based comparisons are sometimes focused on ordinal rankings, so they too examine point-estimates and how they shift rankings between two time periods, facilitating statements such as "For whites, relatively high levels of isolation have declined substantially over the decade 2000–2010" (Clark and Östh, 2018). But in other cases, time-based comparisons examine the rate of change between two places (i.e. the slope in each city's segregation), enabling statements such as "The level of black-white dissimilarity increased sharply during each decade after 1910" (Massey and Hajnal, 1995). Extending temporal comparisons, researchers making space-time comparisons between segregation measures tend to plot the linear trends for each city, compare cross sectional measures between cities, and compare the trendlines between cities to facilitate statements such as "From 2000 to 2010... economic segregation increased in 72 CZs [and] larger metro areas tend to be more segregated than less populous metros" (Acs et al., 2017).

Much like its cousins, space-time comparisons in segregation research also span the globe (van Ham and Tammaru, 2016) and in a variety of spatial and aspatial social science contexts (Clark, 1986b; Blackburn et al., 1993; Johnston et al., 2004; Lichter et al., 2007; Fowler, 2016). But as a methodological amalgamation of spatial and temporal comparisons, space-time comparisons indeed suffer the combined flaws of each. This literature, too, makes clear that segregation measurement strategies are fraught with difficulty since, "at a minimum we would expect satisfactory measures to provide consistent comparisons across place and over time" (Blackburn et al., 1993, p.340) but this is not the case. Even with modern, spatially explicit segregation indices, space-time comparisons are particularly problematic because each urban system has multiple variables changing in concert. Each city experiences changes in its population structure and urban development patterns (and thus, census enumeration units) and there are no methods that permit researchers to decompose the measured differences to understand *which* variable is a larger contributor to the results. Nor are there guidelines that help researchers analyze whether the magnitude in either segregation slopes or point-estimates are meaningful.

Beyond Ordinal Rankings

While it dominates much of the literature, ordinal comparisons are not the only strategy employed by researchers to investigate patterns of urban segregation. Another common strategy is to calculate segregation indices to serve as dependent variables in regression models. For example researchers have examined whether density (Pendall and Carruthers, 2003), land use regulation (Lens and Monkkonen, 2016), or population diversity (Johnston et al., 2004) explain differing levels of segregation in American cities. Recently, Garcia-López and Moreno-Monroy (2018) find the spatial structure, in the form of mono/polycentricity affects measured income segregation in Brazilian cities. These last two studies are especially poignant because they begin to highlight the importance of both demographic structure and spatial structure and their effects on the resulting measurements of segregation in an urban area. The literature makes clear that the geometric size and configuration of the tabulation units on which segregation measures rely affect the resulting indices significantly (Massey, 1978b; Jakubs and Jakubus, 1981; Wong, 2004; Krupka, 2007; Lee et al., 2008; Clark and Östh, 2018)

Regression approaches that attempt to hold constant certain aspects of spatial structure, like development intensity or polycentricity attempt to rectify this situation, but since such approaches also fail to account for other aspects of spatial structure like the total size of a city or the shape and configuration of its infrastructure networks, housing unit makeup, or neighborhood configuration, it is impossible to disentangle the effects of spatial structure from aggregate segregation measures. Thus, rather than control for the effects of spatial structure, we instead *leverage* it to assess how much of the difference between two measures of segregation is attributable to physical layout as opposed to demography.

Our review of the existing work on comparative segregation analytics elucidates two clear gaps in the research. First, when comparing two places, researchers lack a framework for understanding the means through which the difference arises. Existing methods fail to provide information about whether the difference between two segregation measures arises from differences in the physical layout of a city (its spatial structure) or the demographic makeup of its population (its social composition). In the sections to follow, we address this question in detail by developing a novel method for decomposing segregation indices into their spatial and social contributors.

Second, when comparing two places, we lack an *inferential* framework for understanding the scale and scope of segregation, and whether differences between two measures are statistically significant. Among a host of other concerns, an inferential framework requires the proper specification of a testable null hypothesis. In this paper, we do not focus on the development of an inferential framework; to construct our decomposition measures, however, we do require the formulation of a counterfactual. Thus, in the proceeding section, we describe a new computational technique for constructing the counterfactual distributions that we leverage in our decomposition approach.

3 A Framework for Comparative Segregation Analysis

Our analytical framework uses the following structure. Consider Table 1 which reports data for a particular city at one point in time. The rows correspond to the enumeration units (census blocks or tracts), while the second and third columns are associated with each ethnic/racial group. We assume that $n_{i,j}^{a,t}$ is the population of unit $i \in \{1, ..., I\}$ of group $j \in \{x, y\}$ in city a and period t. We, usually, consider group x as being the group of interest (also called the *minority* group). In addition, the marginal and total sums are given by $\sum_j n_{i,j}^{a,t} = n_{i,\cdot}^{a,t}$, $\sum_i n_{i,j}^{a,t} = n_{\cdot,j}^{a,t}$, $\sum_i \sum_j n_{i,j}^{a,t} = n_{\cdot,\cdot}^{a,t}$ which are, respectively, the total population of unit i, total city population of group j and total city population. We also define $\tilde{s}_{i,j}^{a,t} = \frac{n_{i,j}^{a,t}}{n_{i,\cdot}^{a,t}}$ as the share of tract i's population belonging to group j (also called *unit composition*) and $\hat{s}_{i,j}^{a,t} = \frac{n_{i,j}^{a,t}}{n_{\cdot,j}^{a,t}}$ as the share of the city's population in group j that resides in tract i.

We adopt the perspective of Allen et al. (2015) and view segregation as an assignment process that distributes values to the internal cells of Table 1 subject to the row and column constraints. In comparing different cities across space, or the same city over time, it is important to note that not only does the distribution of the values over the internal cells of the table matter but also the marginal row and column distributions. Small overall proportions of minority groups can result in the minority group being unevenly distributed by chance, relative to groups with larger shares of the city's population.

Tract	Group x	Group y	Total
1	$n_{1,x}^{a,t}$	$n_{1,y}^{a,t}$	$n_{1,.}^{a,t}$
2	$n_{2,x}^{a,t}$	$n_{2,y}^{a,t}$	$n_{2,.}^{a,t}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
I	$n_{I,x}^{a,t}$	$n_{I,y}^{a,t}$	$n_{I,.}^{a,t}$
Total	$n_{.,x}^{a,t}$	$n_{.,y}^{a,t}$	$n_{.,.}^{a,t}$

Table 1: Generic structure of a dataset of a given city a in the t period.

		Spatial Structure	
		City 1	City 2
Attribute	City 1	G_A	G_B
Distribution	City 2	G_C	G_D

Table 2: Cross-sectional Decomposition of Segregation Differences

Decomposition

We formulate a general structure that supports the comparative analysis of segregation across two different contexts. Depending on the nature of the context, (spatial, temporal, or spatio-temporal) different formulations arise; the same general structure, however, can be used to identify the key dimensions of each comparison. Two dimensions are relevant for comparative segregation analysis: the distribution and spatial structure, which we define as the allocation of people across the physical space of a study area, which is a function of natural barriers, transportation networks, aggregation units, urban planning, and other elements.

Cross-sectional Segregation Decomposition

Table 2 provides an illustration of these dimensions for a cross-sectional comparison case involving two cities at one point in time. Here interest centers on comparison of the segregation indices measured for City 1 versus City 2, corresponding to the two segregation indices associated with cases A and D in the table. For now, we assume that the chosen segregation index is the Gini coefficient.

The observed difference $\Delta G_{A,D} = G_A - G_D$ may be due to differences in spatial structure as well as differences in the attribute distributions across the two cities. To decompose the observed differences across these dimensions, we adopt a Shapley decomposition approach (Shorrocks, 2013). In formal terms, we define a function:

$$\Delta G = G(S_1, A_1) - G(S_2, A_2) \tag{1}$$

and then define the Shapley contributions of the spatial S and attribute A components, given respectively as C_S and C_A, as:

$$F(S, A) = C_S + C_A = \Delta G \tag{2}$$

with:

$$C_S = \frac{1}{2}\left[G(S_1, A_1) - G(S_2, A_1) + G(S_1, A_2) - G(S_2, A_2)\right] \tag{3}$$

and:

$$C_A = \frac{1}{2}\left[G(S_1, A_1) - G(S_1, A_2) + G(S_2, A_1) - G(S_2, A_2)\right]. \tag{4}$$

Focusing on the spatial component, C_S, in (3), there are two estimates that are obtained. The first holds the attribute distribution constant, and set to that of City 1, while the spatial structure varies between the two cities. In the second estimate, spatial structure varies but the attribute distribution is constant and taken from City 2. The final spatial component is taken as the average of these two estimates. Note that in each of these estimates, there is a counterfactual that is produced and used against the observed Gini index for a particular City. In the first sub-estimate the counterfactual is $G(S_2, A_1)$ which calculates the Gini for a realization where the attribute distribution for City 1 is imposed on the spatial structure of City 2. The difference between the Gini from this counterfactual and that from the observed Gini $G(S_1, A_2)$ is attributed to changing the spatial structure since it is the only component that varies between the two cases. In the second estimate, the counterfactual obtains from imposing the attribute distribution of City 2 on the spatial structure of City 1. Again, only the spatial structure changes. Below we return to discuss how these counterfactuals are constructed, here we focus on the interpretation of the decomposition.

To estimate the Shapley contribution of the attribute distribution, a similar approach is taken in (4) only now the two estimates obtain from holding the spatial structure fixed to that in one of the cities, while allowing the attribute distribution to vary.

Temporal Segregation Decomposition

The comparative analysis of the same city at two points in time can be viewed in a similar fashion as shown in Table 3. Here the difference in the measure of segregation for this city over time is $\delta G_{A,D} = G_A - G_D$, and now the question is how much of the *change* in the city's segregation is due to changes in spatial

		Spatial Structure	
		Period 1	Period 2
Attribute	Period 1	G_A	G_B
Distribution	Period 2	G_C	G_D

Table 3: Temporal Decomposition of Segregation Differences

structure versus changes in its attribute distribution over the two periods. The estimates of the Shapley contributions of the changes in spatial structure and the changes in the city's attribute distribution can be obtained using (3) and (4) only the interpretation changes as the subscript for the arguments to the Gini functions refer to either time period 1 or time period 2.

Counterfactual Distributions

To generate the counterfactual distributions that are used in the Shapley decomposition, we first estimate the tract-level composition of a particular group in each city. Using the notation from Table 1 we use the fact that $\tilde{s}^{1,t}_{i,j}$ is the unit composition of group j in tract i of City 1 in the period t.

Next, we form the cumulative distribution functions (CDF) for these values taken over all the tracts in City 1: $F^{(1)}(\tilde{s}^{1,t}_{i,j})$, and City 2: $F^{(2)}(\tilde{s}^{2,t}_{i,j})$. To create a counterfactual distribution that imposes the attribute distribution of City 2 on the spatial structure of City 1 we take $p^{1,t}_{i,j} = F^{(1)}(\tilde{s}^{1,t}_{i,j})$ and then generate $n^{1,t}_{i,j}|_{attr=2} = {F^{(2)}}^{-1}(p^{1,t}_{i,j})n^{1,t}_{i,.}$, where $attr = 2$ means that this population is calculated given the attributes of City 2. This entire process is done for all tracts of a group in City 1 and the majority group population is given by the difference $n^{1,t}_{i,.} - n^{1,t}_{i,j}|_{attr=2}$. The populations for City 2 are generated analogously.[3]

The intuition behind the counterfactuals is as follows. In Table 2, the counterfactual for case B involves imposition of the unit composition CDF from City 1 on the spatial structure of City 2. This respects the spatial distribution of unit composition rankings in City 2, only the level of the unit composition is taken from the value corresponding to the same rank but in City 1. In other words, the location of tracts with high minority composition follows the distribution from City 2, but the value of the minority share is obtained from City 1. For the second counterfactual, the situation is reversed, where now the value of the minority shares are

[3]This approach can be extended assuming counterfactual composition distributions for both groups (the group of interest and the complementary/majority group). In this case, the counterfactual total population is generated by adding both counterfactual population values.

obtained from the quantile function for City 2 using the observed values of the CDF for each tract's unit share in City 1.

Each counterfactual can then be compared against a different observed case. In comparing the Gini from case B to case A, we are asking how segregation would change if the composition of City 1 was to be imposed on the spatial structure of City 2, instead of City 1's spatial structure. Comparing case D to case B asks the question of how segregation in City 2 would change if its unit composition distribution was replaced with that of City 1.

An alternative way to construct the counterfactual distributions is to use use the unit shares $\hat{s}^{a,t}_{i,j}$. In this approach, four distinct CDFs are built for each group (minority and majority) in each city given by $p^{1,t}_{i,x} = F^{(1,x)}(\hat{s}^{1,t}_{i,x})$, $p^{1,t}_{i,y} = F^{(1,y)}(\hat{s}^{1,t}_{i,y})$, $p^{2,t}_{i,x} = F^{(2,x)}(\hat{s}^{2,t}_{i,x})$ and $p^{2,t}_{i,y} = F^{(2,y)}(\hat{s}^{2,t}_{i,y})$. In this case, we generate each frequency with $n^{1,t}_{i,x}|_{attr=2} = {F^{(2,x)}}^{-1}(p^{1,t}_{i,x})n^{1,t}_{.,x}$, $n^{1,t}_{i,y}|_{attr=2} = {F^{(2,y)}}^{-1}(p^{1,t}_{i,y})n^{1,t}_{.,y}$, $n^{2,t}_{i,x}|_{attr=1} = {F^{(1,x)}}^{-1}(p^{2,t}_{i,x})n^{2,t}_{.,x}$ and $n^{2,t}_{i,y}|_{attr=1} = {F^{(1,y)}}^{-1}(p^{2,t}_{i,y})n^{2,t}_{.,y}$.

There are important differences between to two approaches to generating the counterfactual distributions, and these pertain to which marginal totals from table 1 are respected. In the unit composition approach, total tract population values for the synthetic realizations will respect the observed values, but the tract composition, and thus city wide composition, values will differ from the observed values. In the second approach, the use of the city share distributions will result in the same city wide composition between the synthetic and observed populations, only now the tract/unit values will vary between the synthetic and observed populations.

4 Illustration

Los Angeles vs. New York: Cross-Sectional comparison in 2010

To Illustrate our cross-sectional comparison approach, we use 2010 census data for the metropolitan statistical areas (MSAs) of Los Angeles and New York, and focus on segregation measures for the non-hispanic black population using tract level data. Figure 1 illustrates how the non-hispanic black population is distributed differently between these two MSAs. The maps in (a) and (c) show how this group is distributed in space in their respective metropolitan areas in 2010. The maps in (b) and (c) show the how we would expect non-hispanic blacks to be distributed if we imposed the population structure of the opposing city, but kept existing spatial layout the same. Put differently, the counterfactual example in Figure 1b shows how we would expect New York's non-hispanic black population distribution to look if we replaced its observed tract composition CDF with that of Los Angeles'.

Figure 1c reverses the process to replace Los Angeles' tract composition CDF with that of New York's.

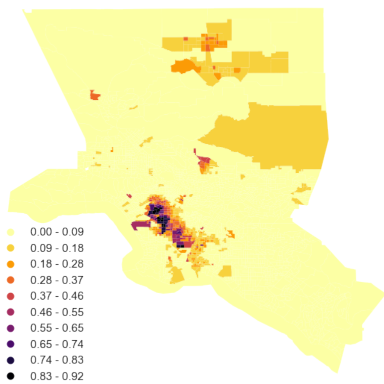
In Los Angeles, shown in subfigure 1a, there is a clear pattern of spatial concentration and unevenness in the racial makeup, in that the non-hispanic black population appears heavily concentrated into a single area of the city. New York, by contrast, shown in subfigure 1d, has a unique pattern that is distinct from Los Angeles, with multiple hotspots of non-hispanic black population, mostly concentrated in Kings County, a portion of Queens and, with less intensity, in the Bronx. In the parlance of regional science, we would argue the structure of segregation for non-hispanic blacks in Los Angeles appears essentially monocentric whereas the structure in NYC is clearly polycentric, a curious reversal of their economic forms. According to their Gini indices, the segregation estimate in Los Angeles was 0.692 and for New York was 0.798.

Given all of these differences, both in terms of spatial context and population compositions, comparing segregation between the two cities poses a considerable challenge. The difference between Ginis for the two cities (-0.106) could be caused by a variety of these factors, and we employ the Shapley decomposition approach described earlier to disentangle them. The first thing to consider for the Shapley decomposition is the cumulative distribution function (CDF) for the black population of each city, shown for reference in Figure 2. New York City is home to a larger population of non-hispanic black residents and, therefore, its distribution reaches the cumulative value of unity only in the far right corner of the graph, whereas Los Angeles, represented by the blue curve, has lower values and reaches unity more rapidly.

To build the counterfactual segregation values to inspect each component in the Shapley decomposition of Table 2, we applied $F^{(NY)}$ in the original composition of Los Angeles and $F^{(LA)}$ in the original composition of NY and constructed counterfactual CDFs distribution that are illustrated in Figure 1 through subfigures 1b and 1c. These "new" simulated distributions are used to fill the Shapley decomposition in Table 4:

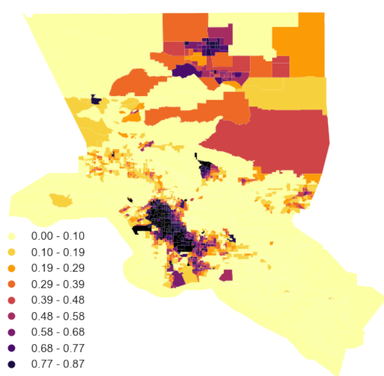
		Spatial Structure	
		Los Angeles	New York
Attribute	Los Angeles	0.692	0.655
Distribution	New York	0.821	0.798

Table 4: Cross-sectional Decomposition of Segregation Differences: Los Angeles and NY

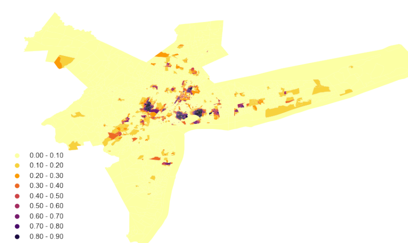


(a) Original census tract composition of Los Angeles (G_A)

(b) Counterfactual distribution: NY space and Los Angeles census tract composition (G_B)



(c) Counterfactual distribution: Los Angeles space and NY census tract composition (G_C)



(d) Original census tract composition of NY (G_D)

Figure 1: non-hispanic black population census tract composition in 2010: Cross-Sectional Comparison.

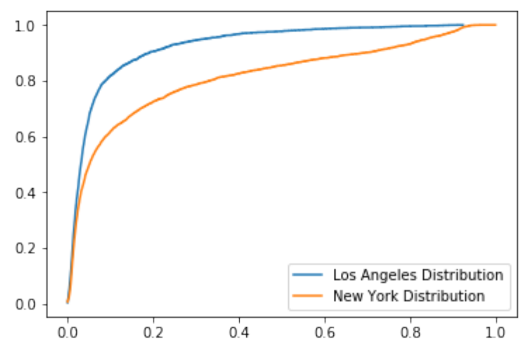


Figure 2: Cumulative Density Functions of non-hispanic black population unit-composition: Cross-Sectional Comparison

The first thing to notice in this decomposition, is that differences among rows are considerably smaller than differences among columns. From Equations 3 and 4 we estimate that the attribute component C_A plays a much more important role in the segregation difference of the two cities since $C_S = 0.030$ and $C_A = -0.136$[4]. In common terms, this result implies that the difference in segregation between Los Angeles and NYC (as measured by the Gini Index) is attributable primarily to the fact that the cities have different shares of residents that identify as black, white, and other races. If instead column differences were greater than row differences, it would imply that physical layout is the greater contributor to measured differences between the cities.

In addition to the relative magnitudes of these components, it is also interesting to explore their directional effects. When holding the attribute distribution constant, a shift to the spatial structure of New York in place of Los Angeles results in a lowering of the segregation index. Focusing on the attribute distribution component, swapping in New York's attribute distribution for that of Los Angeles results in an increase in the segregation index, regardless of the spatial structure.

Los Angeles vs. Los Angeles: Temporal comparison between 2010 and 2000

For temporal comparative segregation analysis, we examine the evolution of Los Angeles between 2000 and 2010 for the same non-hispanic black population. As reported in Table 5, the difference in Gini was -0.040 where, once again, the at-

[4]Assuming counterfactual composition distributions for both groups, the differences obtained were not expressive since $C_S = 0.033$ and $C_A = -0.139$.

tribute component played an important role as $C_S = -0.003$ and $C_A = -0.037$.[5]

		Spatial Structure	
		2010	2000
Attribute	2010	0.692	0.694
Distribution	2000	0.728	0.732

Table 5: Temporal Decomposition of Segregation Differences in Los Angeles: LA in 2010 and LA in 2000

This is to be expected, as the amount of change in the spatial structure of a city over a 10-year period is likely to be dwarfed by demographic change. That being said, the difference in spatial structure between 2000 and 2010, while small, works to reduce segregation (i.e., case B vs. A, or case D vs. C).

Multiple metropolitan regions across US: Cross-Sectional comparison in 2010

Given this decomposition illustration using the context of LA and NY, one might be interested in how this behaves for the rest of the country. We extended this approach to the 50 most populated MSAs of US and decomposed the Gini index for each of the 1225 pairwise comparisons of these MSAs. The values of Gini for all the 50 MSAs sorted can be found in Table 6. In this table, Milwaukee appears the most segregated MSA with the highest Gini of 0.8781 whereas San Jose has the lowest value of 0.3454.

Figures 3 and 4 present the distribution of each of the Shapley components along with the point difference in segregation, respectively, with a density distribution and a violin plot. In these figures, the attribute component is clearly more influential than the spatial component overall as it "dominates" the variability of the point difference. The spatial components typically have values close to zero, whereas the attribute values have considerable variance.

We can look in more detail, however, at each of the comparisons by analyzing the values themselves for the specific MSA pairwise comparisons.[6] We can see some selected results in Tables 7 and 8 where each of these highlights the 30 most

[5]We chose to omit all the details of using the alternative approach of using the counterfactual distributions with the share of each group of each city. However, we highlight here that the results were in accordance with the previous since $C_S = -0.044$ and $C_A = -0.061$ for the cross-sectional example between Los Angeles and NY and $C_S = -0.011$ and $C_A = -0.029$ for the temporal evolution of Los Angeles.

[6]Since we have 1225 point estimation, supplementary materials with all comparisons is available online in this Note: Removed during review to protect author confidentiality Jupyter Notebook.

Metro	Gini	Rank	Metro	Gini	Rank
Milwaukee-Waukesha-West Allis, WI	0.8781	1	Nashville-Davidson–Murfreesboro–Franklin, TN	0.6692	26
Detroit-Warren-Dearborn, MI	0.8681	2	Denver-Aurora-Lakewood, CO	0.6647	27
Cleveland-Elyria, OH	0.8629	3	Richmond, VA	0.6629	28
Chicago-Naperville-Elgin, IL-IN-WI	0.8563	4	Tampa-St. Petersburg-Clearwater, FL	0.6579	29
St. Louis, MO-IL	0.8499	5	Houston-The Woodlands-Sugar Land, TX	0.6373	30
Buffalo-Cheektowaga-Niagara Falls, NY	0.8305	6	Oklahoma City, OK	0.6366	31
Cincinnati, OH-KY-IN	0.8042	7	Dallas-Fort Worth-Arlington, TX	0.6272	32
New York-Newark-Jersey City, NY-NJ-PA	0.7982	8	San Francisco-Oakland-Hayward, CA	0.6263	33
Pittsburgh, PA	0.7811	9	Minneapolis-St. Paul-Bloomington, MN-WI	0.6195	34
Baltimore-Columbia-Towson, MD	0.7806	10	Charlotte-Concord-Gastonia, NC-SC	0.6159	35
Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	0.7788	11	Virginia Beach-Norfolk-Newport News, VA-NC	0.6033	36
Miami-Fort Lauderdale-West Palm Beach, FL	0.7673	12	San Antonio-New Braunfels, TX	0.5839	37
Indianapolis-Carmel-Anderson, IN	0.7671	13	Orlando-Kissimmee-Sanford, FL	0.5835	38
New Orleans-Metairie, LA	0.7605	14	Providence-Warwick, RI-MA	0.5646	39
Columbus, OH	0.7501	15	Sacramento–Roseville–Arden-Arcade, CA	0.5453	40
Rochester, NY	0.7454	16	Seattle-Tacoma-Bellevue, WA	0.5325	41
Memphis, TN-MS-AR	0.7418	17	Portland-Vancouver-Hillsboro, OR-WA	0.5199	42
Louisville/Jefferson County, KY-IN	0.7295	18	Raleigh, NC	0.5183	43
Boston-Cambridge-Newton, MA-NH	0.7182	19	Austin-Round Rock, TX	0.5046	44
Kansas City, MO-KS	0.7160	20	San Diego-Carlsbad, CA	0.4963	45
Washington-Arlington-Alexandria, DC-VA-MD-WV	0.7133	21	Riverside-San Bernardino-Ontario, CA	0.4414	46
Atlanta-Sandy Springs-Roswell, GA	0.7030	22	Phoenix-Mesa-Scottsdale, AZ	0.4139	47
Hartford-West Hartford-East Hartford, CT	0.7025	23	Las Vegas-Henderson-Paradise, NV	0.3842	48
Los Angeles-Long Beach-Anaheim, CA	0.6917	24	Salt Lake City, UT	0.3634	49
Jacksonville, FL	0.6713	25	San Jose-Sunnyvale-Santa Clara, CA	0.3454	50

Table 6: Ranked Gini values for all 50 MSAs

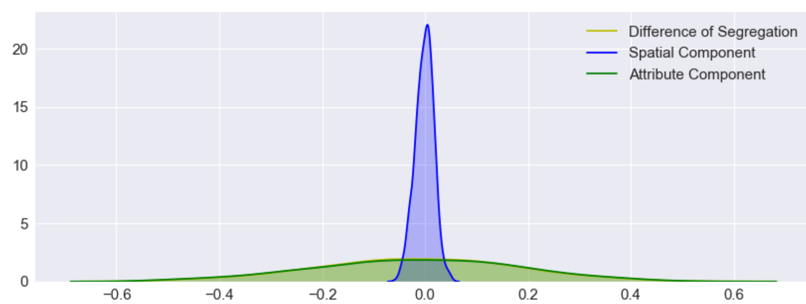


Figure 3: Shapley Components Densities Plot

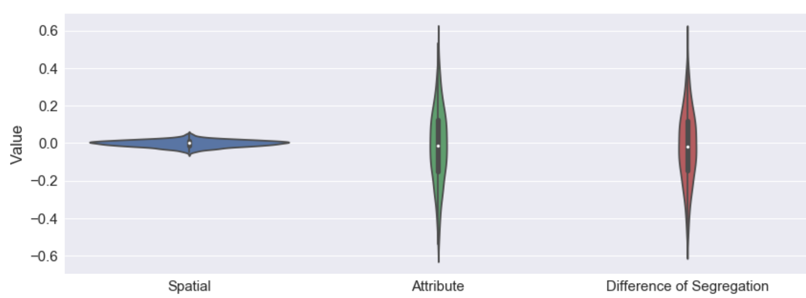


Figure 4: Shapley Components Violins Plot

relevant metropolitan comparison sorted in terms of the magnitude of the spatial and attribute Shapley component, respectively. The first 15 lines of each table represent the lowest values and the last 15 lines the highest values.

Metro A	Metro B	Difference of Segregation	Spatial	Attribute
Kansas City, MO-KS	Los Angeles-Long Beach-Anaheim, CA	0.0244	-0.0587	0.0830
Jacksonville, FL	Los Angeles-Long Beach-Anaheim, CA	-0.0203	-0.0537	0.0333
Orlando-Kissimmee-Sanford, FL	Los Angeles-Long Beach-Anaheim, CA	-0.1082	-0.0513	-0.0569
Kansas City, MO-KS	Denver-Aurora-Lakewood, CO	0.0513	-0.0510	0.1023
Jacksonville, FL	Providence-Warwick, RI-MA	0.1067	-0.0482	0.1549
New Orleans-Metairie, LA	Los Angeles-Long Beach-Anaheim, CA	0.0689	-0.0473	0.1162
Kansas City, MO-KS	Dallas-Fort Worth-Arlington, TX	0.0888	-0.0463	0.1352
Jacksonville, FL	San Jose-Sunnyvale-Santa Clara, CA	0.3260	-0.0461	0.3720
Kansas City, MO-KS	Boston-Cambridge-Newton, MA-NH	-0.0022	-0.0460	0.0438
Jacksonville, FL	Denver-Aurora-Lakewood, CO	0.0066	-0.0451	0.0517
Kansas City, MO-KS	Portland-Vancouver-Hillsboro, OR-WA	0.1961	-0.0448	0.2409
Kansas City, MO-KS	Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	-0.0628	-0.0447	-0.0181
Orlando-Kissimmee-Sanford, FL	Denver-Aurora-Lakewood, CO	-0.0812	-0.0447	-0.0365
Jacksonville, FL	Las Vegas-Henderson-Paradise, NV	0.2871	-0.0443	0.3314
Orlando-Kissimmee-Sanford, FL	San Antonio-New Braunfels, TX	-0.0004	-0.0435	0.0431
⋮	⋮	⋮	⋮	⋮
Salt Lake City, UT	Jacksonville, FL	-0.3079	0.0402	-0.3481
Hartford-West Hartford-East Hartford, CT	Jacksonville, FL	0.0312	0.0405	-0.0093
San Jose-Sunnyvale-Santa Clara, CA	Orlando-Kissimmee-Sanford, FL	-0.2381	0.0415	-0.2796
Las Vegas-Henderson-Paradise, NV	Orlando-Kissimmee-Sanford, FL	-0.1993	0.0421	-0.2414
San Jose-Sunnyvale-Santa Clara, CA	Kansas City, MO-KS	-0.3707	0.0424	-0.4131
Hartford-West Hartford-East Hartford, CT	Kansas City, MO-KS	-0.0135	0.0433	-0.0568
Salt Lake City, UT	Kansas City, MO-KS	-0.3526	0.0443	-0.3969
Raleigh, NC	Orlando-Kissimmee-Sanford, FL	-0.0652	0.0451	-0.1103
Providence-Warwick, RI-MA	Orlando-Kissimmee-Sanford, FL	-0.0189	0.0456	-0.0645
Oklahoma City, OK	Jacksonville, FL	-0.0347	0.0462	-0.0809
Raleigh, NC	Jacksonville, FL	-0.1530	0.0474	-0.2004
Las Vegas-Henderson-Paradise, NV	Kansas City, MO-KS	-0.3318	0.0477	-0.3796
Providence-Warwick, RI-MA	Kansas City, MO-KS	-0.1514	0.0487	-0.2001
Oklahoma City, OK	Kansas City, MO-KS	-0.0794	0.0508	-0.1302
Raleigh, NC	Kansas City, MO-KS	-0.1977	0.0526	-0.2503

Table 7: Top 15 metropolitan regions with **lowest** Shapley spatial component and top 15 metropolitan regions with **highest** Shapley spatial component

The metropolitan areas of Kansas and Los Angeles were the pair which had the lowest spatial component of $C_S = -0.0587$ whereas the highest value was due to the Raleigh and Kansas of $C_S = 0.0526$. We note that by analyzing each component in isolation, the signal shall not be considered very important as it depends exclusively on the order in which the comparison is being decomposed.

Although these are the highlighted differences for the spatial component, it is clear it is the attribute component that it is playing the most important share in the decomposition, especially for the Raleigh-Kansas comparison where $C_A = -0.2503$ and it is illustrated in Figure 5[7]. In this figure, we can check that the spatial extent is very different but not being able to overcome the difference imposed by the different composition that is swapped under the hood of the

[7] Since in some cases the magnitude of the composition values of the pairwise comparison is very different, we chose to present the maps with individual gradient colors to capture the visual spatial behavior of each MSA.

counterfactual of each distribution. We can also see that the spatial distribution is different in terms of the area of the census tracts, but this is not captured in the construction of the Gini index used here.

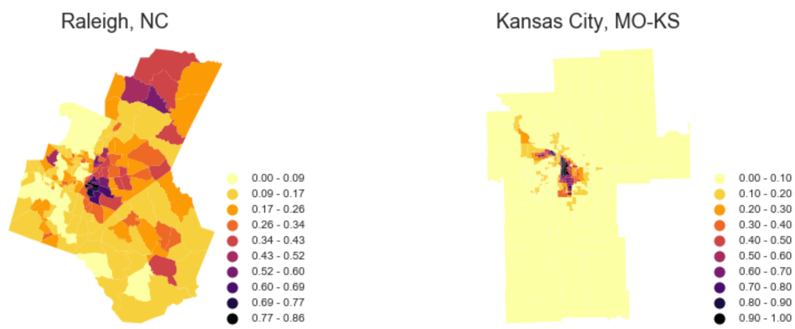


Figure 5: non-hispanic black population census tract composition in 2010 for the highest spatial component ($C_S = 0.0526$) in all pairwise comparisons: Raleigh vs. Kansas

As mentioned before, in this illustration the attribute component appears more relevant importance in terms of the magnitude of the decomposition. Table 8 presents the principal comparison that generated the lowest values and highest values, respectively, in the first and last lines. It is possible to see that San Jose-Sunnyvale-Santa Clara, CA, and Detroit-Warren-Dearborn, MI, had the lowest value of $C_A = -0.5363$ whereas the comparison between Milwaukee-Waukesha-West Allis, WI and San Jose-Sunnyvale-Santa Clara, CA the highest value of $C_A = 0.5304$. To illustrate how this highlighted comparison is expressed graphically, Figure 6 depicts the former comparison where the two metropolitan regions have composition magnitudes at a very different scale. San Jose has non-hispanic black population composition values ranging from 0% to 14% and Detroit from 0% to 100%. Although the difference in their spatial structure is visible plainly, the contrast between the cities' demographic structure exceeds dramatically the spatial distinctions between them, resulting in the most extreme case in the data.

In this data, usually, the Shapley attribute component "dominates" the spatial one, but it is also of interest to investigate whether there are cases where the spatial component is "more important" (e.g. is larger in absolute magnitude). To examine these instances, we select from our set comparisons where $|C_S| > |C_A|$ and sort the data according to the difference given by $|C_S| - |C_A|$ to check whether the comparison produced a spatial component more relevant than the attribute

Metro A	Metro B	Difference of Segregation	Spatial	Attribute
San Jose-Sunnyvale-Santa Clara, CA	Detroit-Warren-Dearborn, MI	-0.5227	0.0136	-0.5363
San Jose-Sunnyvale-Santa Clara, CA	Chicago-Naperville-Elgin, IL-IN-WI	-0.5110	0.0191	-0.5301
San Jose-Sunnyvale-Santa Clara, CA	Cleveland-Elyria, OH	-0.5176	0.0065	-0.5241
Salt Lake City, UT	Milwaukee-Waukesha-West Allis, WI	-0.5146	0.0051	-0.5198
San Jose-Sunnyvale-Santa Clara, CA	St. Louis, MO-IL	-0.5045	0.0147	-0.5193
Salt Lake City, UT	Detroit-Warren-Dearborn, MI	-0.5046	0.0146	-0.5192
Salt Lake City, UT	Chicago-Naperville-Elgin, IL-IN-WI	-0.4929	0.0212	-0.5141
Salt Lake City, UT	Cleveland-Elyria, OH	-0.4995	0.0143	-0.5138
Salt Lake City, UT	St. Louis, MO-IL	-0.4865	0.0182	-0.5046
Las Vegas-Henderson-Paradise, NV	Detroit-Warren-Dearborn, MI	-0.4839	0.0148	-0.4987
Las Vegas-Henderson-Paradise, NV	Chicago-Naperville-Elgin, IL-IN-WI	-0.4722	0.0228	-0.4949
Las Vegas-Henderson-Paradise, NV	Cleveland-Elyria, OH	-0.4787	0.0131	-0.4919
Las Vegas-Henderson-Paradise, NV	St. Louis, MO-IL	-0.4657	0.0193	-0.4850
Salt Lake City, UT	Buffalo-Cheektowaga-Niagara Falls, NY	-0.4670	0.0063	-0.4733
San Jose-Sunnyvale-Santa Clara, CA	New York-Newark-Jersey City, NY-NJ-PA	-0.4528	0.0181	-0.4710
⋮	⋮	⋮	⋮	⋮
New Orleans-Metairie, LA	Las Vegas-Henderson-Paradise, NV	0.3764	-0.0354	0.4118
Rochester, NY	Salt Lake City, UT	0.3820	-0.0300	0.4120
Memphis, TN-MS-AR	San Jose-Sunnyvale-Santa Clara, CA	0.3964	-0.0156	0.4120
St. Louis, MO-IL	Riverside-San Bernardino-Ontario, CA	0.4085	-0.0061	0.4145
Cleveland-Elyria, OH	Riverside-San Bernardino-Ontario, CA	0.4215	0.0036	0.4179
Rochester, NY	San Jose-Sunnyvale-Santa Clara, CA	0.4001	-0.0215	0.4216
Milwaukee-Waukesha-West Allis, WI	Riverside-San Bernardino-Ontario, CA	0.4366	0.0115	0.4251
St. Louis, MO-IL	Phoenix-Mesa-Scottsdale, AZ	0.4360	-0.0086	0.4445
New Orleans-Metairie, LA	San Jose-Sunnyvale-Santa Clara, CA	0.4152	-0.0316	0.4468
Cleveland-Elyria, OH	Phoenix-Mesa-Scottsdale, AZ	0.4490	0.0003	0.4487
Buffalo-Cheektowaga-Niagara Falls, NY	Las Vegas-Henderson-Paradise, NV	0.4463	-0.0049	0.4511
Milwaukee-Waukesha-West Allis, WI	Phoenix-Mesa-Scottsdale, AZ	0.4642	0.0081	0.4561
Buffalo-Cheektowaga-Niagara Falls, NY	San Jose-Sunnyvale-Santa Clara, CA	0.4851	-0.0034	0.4885
Milwaukee-Waukesha-West Allis, WI	Las Vegas-Henderson-Paradise, NV	0.4939	-0.0032	0.4971
Milwaukee-Waukesha-West Allis, WI	San Jose-Sunnyvale-Santa Clara, CA	0.5327	0.0023	0.5304

Table 8: Top 15 metropolitan regions with **lowest** Shapley attribute component and top 15 metropolitan regions with **highest** Shapley attribute component

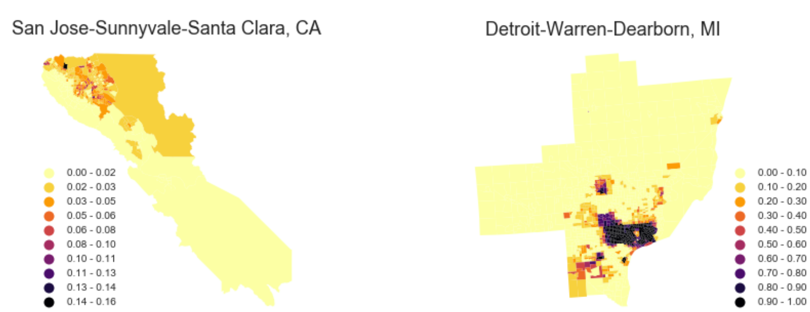


Figure 6: non-hispanic black population census tract composition in 2010 for the lowest attribute component ($C_A = -0.5363$) in all pairwise comparisons: San Jose vs. Detroit

component. We identify 67 cases where $|C_S| > |C_A|$ and the highest value of $|C_S| - |C_A|$ is illustrated in Figure 7. This figure depicts the comparison between Orlando-Kissimmee-Sanford, FL, and Dallas-Fort Worth-Arlington, TX, where the non-hispanic black population composition is given by the color classes scale. It is clear that although the spatial structure is different between the two cities, the demographic composition is very similar, ranging from values of 0% to values around 95%-96%. In this case, therefore, it is the spatial component which is responsible for contributing a greater difference in the measured Gini index between the two cities because it is the configuration of their census tracts that differs more than their population makeup.

Metro A	Metro B	Spatial Absolute Share	Attribute Absolute Share
Buffalo-Cheektowaga-Niagara Falls, NY	Portland-Vancouver-Hillsboro, OR-WA	0.0000	1.0000
Milwaukee-Waukesha-West Allis, WI	Seattle-Tacoma-Bellevue, WA	0.0001	0.9999
Nashville-Davidson–Murfreesboro–Franklin, TN	St. Louis, MO-IL	0.0006	0.9994
Cleveland-Elyria, OH	Phoenix-Mesa-Scottsdale, AZ	0.0007	0.9993
Virginia Beach-Norfolk-Newport News, VA-NC	St. Louis, MO-IL	0.0010	0.9990
Louisville/Jefferson County, KY-IN	Charlotte-Concord-Gastonia, NC-SC	0.0014	0.9986
Cincinnati, OH-KY-IN	San Diego-Carlsbad, CA	0.0014	0.9986
Milwaukee-Waukesha-West Allis, WI	Dallas-Fort Worth-Arlington, TX	0.0017	0.9983
Virginia Beach-Norfolk-Newport News, VA-NC	Houston-The Woodlands-Sugar Land, TX	0.0018	0.9982
Sacramento–Roseville–Arden-Arcade, CA	Dallas-Fort Worth-Arlington, TX	0.0018	0.9982
Indianapolis-Carmel-Anderson, IN	Atlanta-Sandy Springs-Roswell, GA	0.0019	0.9981
Las Vegas-Henderson-Paradise, NV	Minneapolis-St. Paul-Bloomington, MN-WI	0.0019	0.9981
Cincinnati, OH-KY-IN	Charlotte-Concord-Gastonia, NC-SC	0.0020	0.9980
Nashville-Davidson–Murfreesboro–Franklin, TN	Baltimore-Columbia-Towson, MD	0.0021	0.9979
Phoenix-Mesa-Scottsdale, AZ	Miami-Fort Lauderdale-West Palm Beach, FL	0.0024	0.9976
⋮	⋮	⋮	⋮
Virginia Beach-Norfolk-Newport News, VA-NC	Dallas-Fort Worth-Arlington, TX	0.8433	0.1567
Indianapolis-Carmel-Anderson, IN	Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	0.8622	0.1378
New Orleans-Metairie, LA	Pittsburgh, PA	0.8732	0.1268
Hartford-West Hartford-East Hartford, CT	Nashville-Davidson–Murfreesboro–Franklin, TN	0.8910	0.1090
Boston-Cambridge-Newton, MA-NH	Atlanta-Sandy Springs-Roswell, GA	0.8983	0.1017
Orlando-Kissimmee-Sanford, FL	Dallas-Fort Worth-Arlington, TX	0.8985	0.1015
Oklahoma City, OK	Virginia Beach-Norfolk-Newport News, VA-NC	0.9094	0.0906
Virginia Beach-Norfolk-Newport News, VA-NC	San Francisco-Oakland-Hayward, CA	0.9119	0.0881
Tampa-St. Petersburg-Clearwater, FL	Los Angeles-Long Beach-Anaheim, CA	0.9264	0.0736
Milwaukee-Waukesha-West Allis, WI	Detroit-Warren-Dearborn, MI	0.9367	0.0633
Orlando-Kissimmee-Sanford, FL	Minneapolis-St. Paul-Bloomington, MN-WI	0.9374	0.0626
Providence-Warwick, RI-MA	Sacramento–Roseville–Arden-Arcade, CA	0.9413	0.0587
Rochester, NY	Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	0.9498	0.0502
Memphis, TN-MS-AR	Kansas City, MO-KS	0.9501	0.0499
Charlotte-Concord-Gastonia, NC-SC	Dallas-Fort Worth-Arlington, TX	0.9805	0.0195

Table 9: Top 15 metropolitan regions with **lowest** Shapley spatial absolute share and top 15 metropolitan regions with **highest** Shapley spatial absolute share

An alternative way to check the relative importance of one component over another is to inspect its relative absolute value over the sum of both absolute values. In formal terms, one can use $\frac{|C_S|}{|C_S|+|C_A|}$ and $\frac{|C_A|}{|C_S|+|C_A|}$ as the *spatial absolute share* and *attribute absolute share*, respectively, such as presented in Table 9. For this case, however, the results prove slightly different in the MSAs of the top comparison, but still indicating that the composition magnitude plays the most important role in this decomposition.

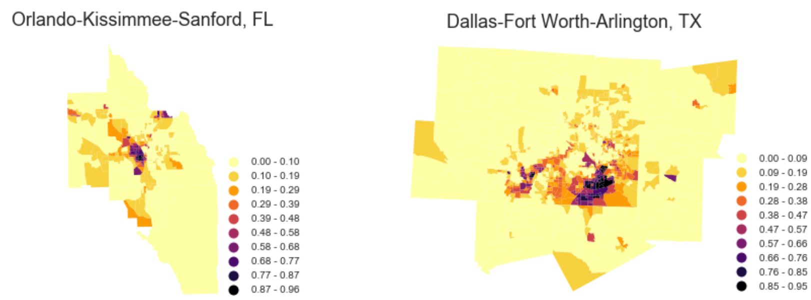


Figure 7: non-hispanic black population census tract composition in 2010 for the highest spatial component over attribute component in absolute values ($C_S = -0.039$ and $C_A = -0.004$) in all pairwise comparisons: Orlando vs. Dallas

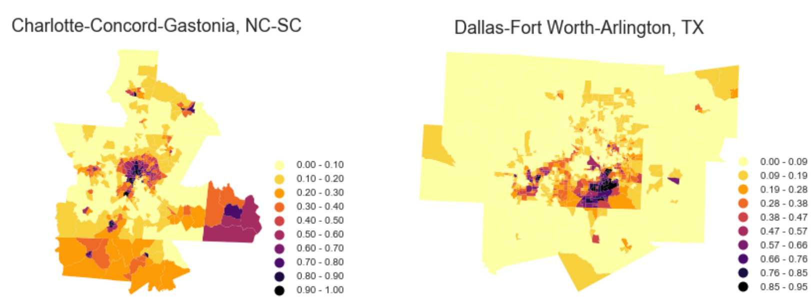


Figure 8: non-hispanic black population census tract composition in 2010 for the highest spatial absolute share (98.05%) in all pairwise comparisons: Charlotte vs. Dallas

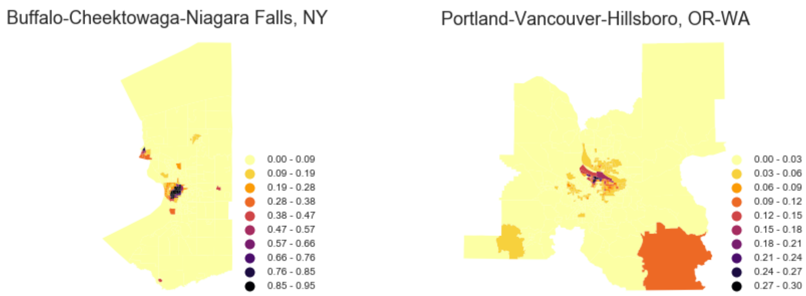


Figure 9: non-hispanic black population census tract composition in 2010 for the lowest spatial absolute share (< 0.01%) in all pairwise comparisons: Buffalo vs. Portland

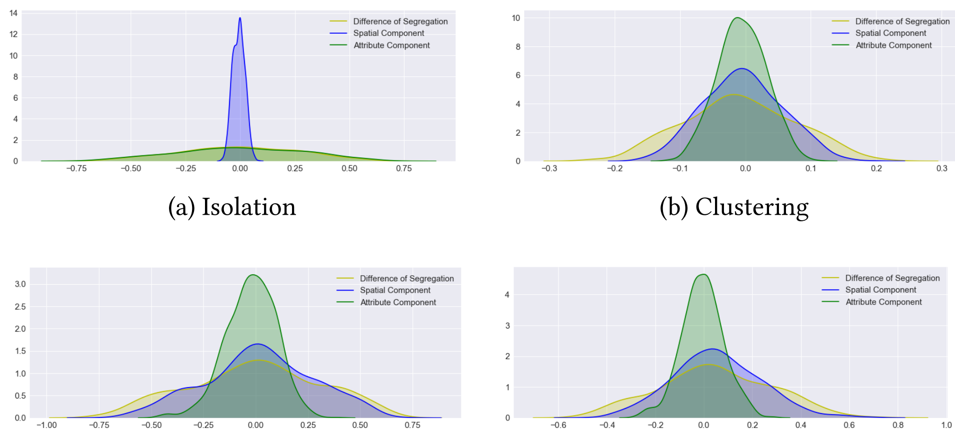
Decomposition under different dimensions of segregation

The Gini index used in the previous section is a measure that assesses the degree of **evenness** of a considered group in a given society. However, different dimensions of segregation can be assessed through different indexes and, according to Massey and Denton (1988), segregation can be considered to have five dimensions: evenness, isolation, clustering, concentration and centralization. Therefore, to check if the interpretation of the results holds for the application, it is of interest to inspect how some indexes behave for each of these dimensions in the Shapley decomposition introduced in section 3.

In this robustness inspection, we chose to use the Isolation index (xPx), the Relative Clustering index (RCL), the Relative Concentration index (RCO) and the Relative Centralization index (RCE) from (Massey and Denton, 1988). The results, given by densities of each component for every MSAs pairwise comparison in the illustration, are present in Figure 10.

From Figures 10 and 3, the differences between the dimensions are clear.

The variance of difference of segregation under the evenness and isolation dimensions can be explained by difference in the variance in the attribute component, whereas for clustering, concentration and centralization the distributions are more mixed, but indicate that the spatial component is more important to these dimensions, since the blue density curve (spatial component density) is closer to the yellow (difference of segregation). The direct reason of this results can be that the clustering, concentration and centralization dimensions are, by definition, spatial. That is, the spatial context is always taken into consideration in the construction of the chosen index, while for evenness and isolation not



(a) Isolation (b) Clustering

(c) Concentration (d) Centralization

Figure 10: Density of the Shapley components of all 1225 paiwise comparisons of the illustation for other dimensions of segregation.

necessarily. In this case, the Gini index and the Isolation index do not take into consideration space for their calculations and, therefore, space naturally may not play an important role in the Shapley decomposition.

These results reinforce the inherent difficulty facing researchers working to develop a comparative segregation framework. This possible sensitivity of how point difference of segregation measures between cities being due to different source of variations poses a challenging feature to handle that imposes a deeper analysis.

Conclusion and Discussion

This study is an attempt of addressing comparative segregation analytics with a novel approach combining counterfactuals distributions and Shapley decompositions. After covering and discussing a literature review on comparing measures of segregation through different examples highlighting the challenges and opportunities in the field, we formalize our mathematical framework and illustrate it with an extensive comparative study for the 50 largest Metropolitan Statistical Areas of US using census tract data.

Our generic approach can be used in any context where the objective is to perform comparative segregation for either space or time or any combination of both. However, one of the challenging steps is that the current decomposition relies on counterfactual distributions of each spatial context under study. In this regard, we discuss that this framework can present different ways of imposing the social composition of one city over another highlighting that the CDF can be used for either spatial unit composition or city compositions. However, although this is a point of concern for this kind of analysis, the Shapley components showed to be relatively robust to the counterfactual approach chosen.

For our illustration using the Gini index, the composition attribute proved to be more relevant than the space attribute when it is used to compare two different spatial contexts. This characteristic persisted in most of our scenarios when either comparing Los Angeles versus New York, performing temporal evolution comparison of Los Angeles or when comparing multiple cities pairwisely. One of the possible reasons behind this is the nature of Gini. Since this index measures the degree of unevenness, it can be affected intensively by the structure of attribute composition and, therefore, more affected by different shapes of CDFs. Given that, one of our key findings is that the decomposition importance can vary depending on the index used that can reflect different dimensions of segregation such as isolation, clustering, concentration or centralization.

Our empirical examples show that in major American cities, the difference in Gini measures is typically due to differences in population structure rather than physical layout Results from pairwise comparisons among the 50 largest MSAs the U.S. showed that differences between cities measured by segregation indices that capture the dimensions of evenness and isolation are due typically to variance in the city's population structure. For the dimensions of clustering, concentration and centralization, however, the city's spatial configuration usually explains the difference. This could suggest that segregation is primarily driven by migration or demography in American cities, but this could demonstrate just how weak the Gini index is for capturing the spatial configuration of segregation. In future work, we plan to explore how these patterns vary for a broad collection of spatially explicit segregation indices with differing definitions of space, including multiscalar segregation profiles (Reardon et al., 2006; Fowler, 2016). We also plan to examine how our results may differ when extending our Shapley decomposition approach to measures of multigroup segregation.

Our illustration also highlights some particularly interesting point comparisons, particularly the differences revealed in Orlando vs. Dallas, and Charlotte vs. Dallas. Both of these comparisons are between sprawling southern cities, suggesting interesting divergence among places that otherwise share a common history, political culture, and demographic structure. Visually, the difference between Charlotte vs. Dallas appears much larger than Orlando vs. Dallas, suggesting differences in Shapley absolute shares may be the more intuitive statistic. Furthermore, these results raise questions regarding the American South's legacy of institutionalized racism, and the way land use policy could help foster more integrated cities. If the revealed difference between Charlotte and Dallas arises through land policy and allocation rather than an artifact of each city's census tract geometry and topology this will be a critically important finding because it implies that governments can use land policy to exercise some degree of control over prevailing segregation patterns. Such a finding would have dramatic implications for the future of urban policy and planning, particularly in the context of Affirmatively Furthering Fair Housing. Another avenue for further research, thus, will be to explore approaches that attempt to explain differences in Shapley spatial components by regressing them on elements of urban form, such as the number, average size, and total population of the city's enumeration units.

Much still has to be done for comparative segregation. An important extension to this work is the development of an inferential component to our decompositional framework. As we mentioned earlier, we see the question of segregation measurement as reflecting a reallocation mechanism, and adopting the bootstrap approach of Allen et al. (2015) but applied to our counterfactual distributions seems like a promising direction in this regard. Additionally, other computationally based approaches to inference such as random labeling (Sastre Gutierrez and Rey, 2013), and random spatial permutations (Anselin, 1995) can be explored to perform comparative inference given a proper specification of a testable null hypothesis mentioned earlier. Finally, the spatial component of the decompositional framework can be revisited by drawing upon analytics from exploratory spatial data analysis (Rey and Arribas-Bel, 2016) and spatial ecology (Fortin and Dale, 2014) with the goal of unmasking deeper insights about the role of spatial structure in segregation dynamics.

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